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WAVE EXPANDER

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A schematic description of the design of a wave expander is presented. A mathematical model of the operation process has been substantiated. The operational characteristics of a reciprocating wave expander have been obtained within the framework of the numerical integration of the system of nonstationary gas-dynamics equations in a one-dimensional formulation. The stages of the operation process have been considered in detail. The dependences of the gasdynamic quantities on the time in the neighborhood of the rotating valve and the oscillating piston have been obtained for the steady-state operating cycle of the wave expander. The expander proposed has been compared to reciprocating expanders of classical design.

Various technical devices are used for artificial cooling of a medium. They are classified, first of all, by their purpose as refrigeration, liquefaction, combined, and gas-separation ones. Each of the fields of application imposes its own requirements on the temperature, refrigerating capacity, mass, dimensions, expenditure of energy, starting time, reliability, and other parameters of the technical devices used.

Reciprocating gas-expansion machines or reciprocating expanders in which the compressed-gas energy is converted directly to work due to the gas-pressure forces are considered to be technically perfect. In such machines, the gas-pressure forces are counterbalanced accurate to an infinitely small value by the drag forces of the brake. Theoretically, the process of expansion is equilibrium and approaches, to a certain extent, the isentropic process. The gasdynamic processes occurring in the sleeve of the cylinder of a reciprocating gas-expansion machine of conventional design are described fairly well in the literature [1, 2]. A theoretical analysis and experiment show that for expanders of classical type (with admission and release valves) the adiabatic efficiency falls in the range of 0.7 to 0.9; however, to obtain such an efficiency it is necessary to impose severe restrictions on the design of the machine and on its parameters.

An attempt to do away with operation within the framework of equilibrium processes led to the development of a series of machines with nonequilibrium expansion of the working substance and the transfer of energy (in the form of heat as a rule to the environment). Aerodynamically, machines of this type are less perfect. This, first of all, concerns the class of machines operating by the Gifford–McMahon cycle and pulsatory cryogenerators in the form of pulsatory tubes, pulsatory gas coolers and Whitley–Swift–Migliori acoustic heat engines. Another class of gas-expansion machines that improve markedly the characteristics of the throttling process is based on the so-called machine-free method of producing refrigeration in wave cryogenerators. In this case, the gas is expanded under the conditions of steady-state flow in which, however, there are regions where a nonstationary wave structure is generated.

The operation of such machines is based on the Hartmann effect (oscillation of a compression shock and generation of a wave process as a result of the inflow of an annular underexpanded jet into any closed cavity) and the Shprenger effect (heating of the closed end of the resonance tube in wave cryogenerators). Excitation of resonance oscillations and their quantitative characteristics are determined by a number of design parameters that must be mutually consistent. For the attained degree of perfection, the mean adiabatic efficiency of wave cryogenerators is 0.12–0.18 and reaches 0.2–0.25 in certain regimes.

The wave expander proposed combines the elements and principles of operation of different devices. Here, as in the classical reciprocating expander, there is a cylinder and a moving piston. Of the principles of operation of a wave cryogenerator, generation of an acoustic wave is used, and admission and release of the gas are technically implemented with the use of the valve mechanism of a pulsatory cooler.

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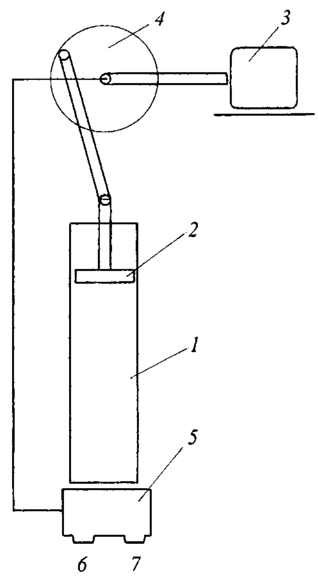


Fig. 1. Diagram of a wave expander.

The diagram of the wave expander is presented in Fig. 1. The expander consists of a cylinder 1, in which piston 2 connected with braker 3 by the driving mechanism 4 executes small-amplitude oscillations. The driving mechanisms also provides rotation of the valve 5 that provides the access of the high-pressure 6 and low-pressure 7 channels to the free end of the cylinder during the cycle.

The processes in the cylinder can be divided into four stages: 1) admission of the cooled gas from the high-pressure channel through the valve into the cylinder, 2) retardation of the admitted gas near the end wall of the valve, 3) release of the admitted gas from the cylinder through the valve to the low-pressure channel, and 4) motion of the gas in the cylinder, as at the retardation stage, near the end wall in a closed volume.

The wave properties of the system are provided by the fact that the frequency of feeding of the gas into the cylinder corresponds to one resonance eigenfrequency of the cylinder. Moreover, the piston oscillations are synchronized with the instants of admission but have a phase shift such that the work of the gas is positive at the instant the shock wave reaches the piston surface.

Let us consider the characteristic stages of the gasdynamic processes occurring inside the cylinder. We assume that the system is adiabatically closed, i.e., there is no heat transfer through the piston and the cylinder walls. The mechanism of action of the valve may be represented as the process of opening and closing of a transversely withdrawn diaphragm.

At the stage of admission of the gas from the high-pressure channel to the cylinder, the diaphragm opens and there arises a shock wave propagating inside the cylinder toward the piston. The so-called contact surface used as the interface between the admitted gas and the gas found in the cylinder moves in the wake of it but with a smaller velocity. The stage of admission of the gas is completed when the diaphragm closes. As a result, the gas in the cylinder is isolated from the high-pressure and low-pressure channels and the stage of retardation of the admitted gas near the end wall of the cylinder in the rarefaction wave begins. In this case, the pressure and temperature of the gas decrease. It is noteworthy that in the process of acceleration of the gas in the rarefaction wave (in the process of its feeding into the cylinder) and in its retardation in the rarefaction wave near the end wall of the valve (inside the cylinder) there occurs motion within the framework of a nearly isentropic process; this enables us to single out reciprocating expanders as the thermodynamically most perfect cryogenerators.

At the stage of release of the admitted gas from the cylinder to the low-pressure channel, the diaphragm connecting the cylinder with the low-pressure channel opens. In the steady-state regime of operation of the device, the contact surface returns to the initial position. At the final stage of the cycle, the diaphragm separates the cylinder from the admission and release channels once again.

The quantitative characteristics of the mass balance in the system in each cycle are expressed in the following manner:

$$\int_0^{\tau} \rho u dt = 0 . \tag{1}$$

In this case, the energy balance on the control valve surface is not retained (the temperatures of the admitted gas and the gas released from the device are different). It can be provided in the cylinder by two methods: if heat is removed from the system or if the gas in the system does positive work. In the present design, two possibilities can be realized in principle.

Let us consider the variant in which positive work is done. This case is attractive for two reasons: first, the processes of heat exchange of the gas with the cylinder walls, excluded from consideration, simplify markedly the analysis of the operation of the device; second, taking account of heat exchange only improves the situation in the sense that it decreases the average temperature of the gas in the cylinder, which exceeds markedly the temperature of the cooled gas.

Unlike the classical expander where all the occurring processes are close to the equilibrium ones, a wave structure is generated here. In this case, during the cycle, the pressures vary within wide limits in the vicinity of the piston in the cylinder, and one can select such a law of piston motion for which the gas does positive work during the cycle. The energy balance will be realized in the system on condition that during the cycle the total-enthalpy flux through the valve structure coincides quantitatively with the work of the piston:

$$\int_0^{\tau} H\rho u dt = \int_0^{\tau} P u_{xp} dt . \tag{2}$$

One would expect that for the selected geometric and operating parameters of the design, the system will come to the dynamic equilibrium state in which conditions (1) and (2) will be fulfilled. By the instant of beginning of the next cycle, there arises a certain equilibrium distribution of the gasdynamic quantities, which differs from the initial state at the instant of start-up of the machine but occurs over and over with time.

By way of example, we consider the design with the following parameters: length of the cylinder 4.0 m, length of the connecting rod 0.5 m, and pass of the piston, 0.3 m. The parameters of the gas in the high-pressure channel are as follows: P = 0.4 MPa, T = 300 K. The gas pressure in the low-pressure channel is 0.1 MPa. A two-atomic gas with a molar mass of μ = 29 kg/kmole is used as the working substance. For the selected geometric parameters and temperature level, the circular frequency providing the first harmonic of near-resonance oscillations is 4500 rpm, the cyclic frequency is 75 Hz, and the duration of the cycle is 0.013333 sec. The duration of different stages of operation of the valve can be selected arbitrarily. In particular, it was assumed that the time of admission is about 14%, the time of retardation is 13%, and the time of release is 53% of the duration of the cycle. The motion of the gas after the closing of the low-pressure channel to the beginning of the next cycle takes 20% of the time of the cycle. To simplify the analysis of the state of the gasdynamic quantities in the cylinder, we assume that the diaphragm closes and opens instantaneously.

Estimation of the characteristic Reynolds numbers enables us to perform mathematical modeling of the gas state in the cylinder within the framework of the system of nonstationary gas-dynamics equations. Taking account of the linear dimensions of the structure and the assumption that the diaphragm closes and opens instantaneously simplifies the description to a one-dimensional formulation.

The nonstationary gas-dynamics equation in divergent form for a deformable control volume may be written as follows:

$$\frac{\partial U}{\partial t} + \frac{\partial F}{\partial x} = 0 . \tag{3}$$

To close the system of gas-dynamics equations, we write the caloric and thermodynamic equations in the form (h = $h(T)$)

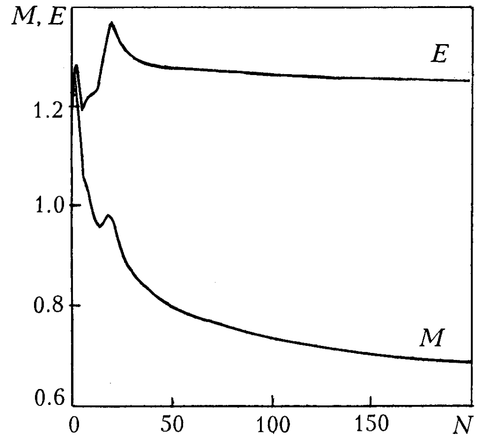
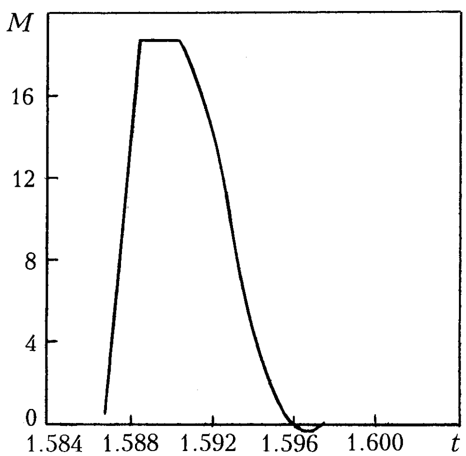


Fig. 2. Dependence of the integral balance of the mass M and the total energy E in the cylinder on the number of complete cycles from the instant of start-up of the wave expander from the state of rest.



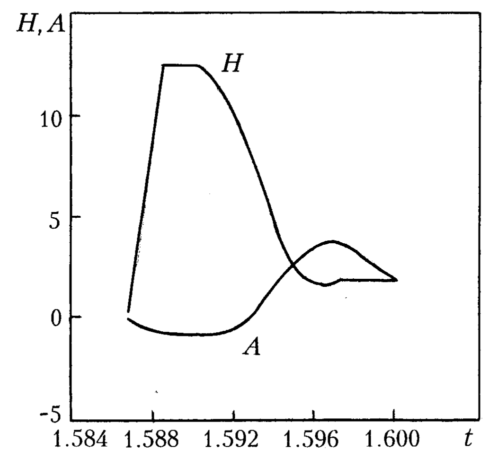


Fig. 3. Change in the mass in the cylinder during the cycle. M, %; t, sec.

Fig. 4. Change in the total-enthalpy flux H in the valve and work A done by the gas on the piston during the cycle. H and A, %; t, sec.

$$H = h + u^2/2\,, \quad P\mu = \rho RT\,. \tag{4}$$

The boundary conditions for Eq. (3) at a penetrable boundary are formulated depending on the direction and value of the velocity: the values of the parameters in the high-pressure channel are used in the case of a subsonic flow (coming into the cylinder) at the boundary and the values of the parameters in the low-pressure channel are used for the subsonic flow coming out of the cylinder. In the case of a possible supersonic flow of the gas out of the cylinder, the boundary conditions are not set for the system of hyperbolic equations in accordance with the theory of characteristic properties. It is assumed that flow is absent at impenetrable boundaries.

Equations (3) and (4) were solved with the use of finite-difference methods. Integration of Eq. (3) was performed within the framework of a modified Godunov two-step scheme of second order of accuracy in time and space [3].

The problem was solved from the instant at which the gas in the cylinder was immobile and the values of the pressure and the temperature were coincident with the values in the low-pressure channel. The evolution of the integral balance of mass and of the total energy from cycle to cycle is presented in Fig. 2. The running values of the mass and the total energy are assigned to their values at the initial instant of time. In particular, by the 200th cycle, the relative error for Eq. (1) was 0.03%. By this time, the relative error for Eq. (2) was 0.003%. As for the character

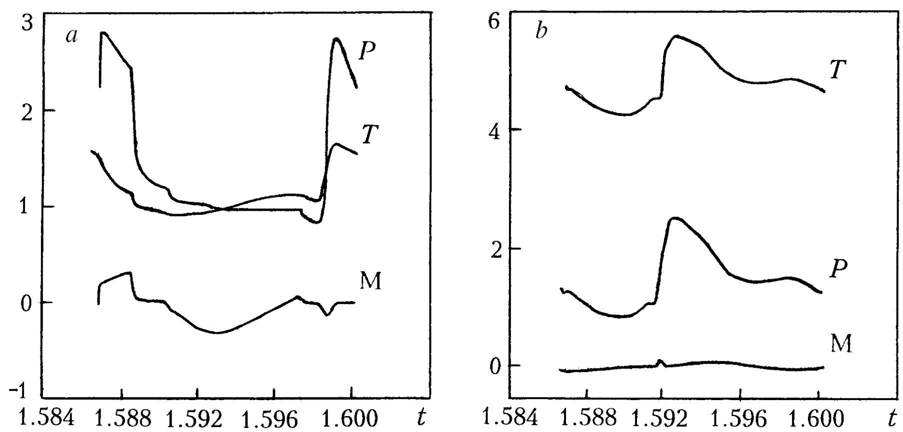


Fig. 5. Change in the pressure, temperature, and Mach number in the cylinder in the neighborhood of the valve (a) and the piston (b) during the cycle.

of change of the functions, the following remarks can be made: a) the initial increase in the integral values is due to the filling of the cylinder with the gas, which leads to an increase in the average pressure; b) the subsequent decrease in the integral values is due to the heating of the gas in the cylinder and the decrease in its density. The slow dynamics of mass stabilization in the cylinder is explained by the formation of the temperature profile in the space.

We analyze some properties of the solution obtained in the neighborhood of the valve and the piston. Figure 3 shows the dynamics of the mass rate of the gas flow through the valve during the cycle. It is clearly seen that the stage of admission of the cooled gas from the high-pressure channel to the cylinder gives way to the stage at which the mass is constant in the cylinder. Next we have the stage of release of the admitted gas from the cylinder to the low-pressure channel and the final stage where the gas in the cylinder moves, as at the stage of retardation near the end wall, in the closed space. The change in the total-enthalpy flux in the valve during the cycle, which is associated with the mass balance, is shown in Fig. 4, where the smooth curve shows the work of the gas on the piston. The characteristics presented in Figs. 3 and 4 correspond to the values of the mass and the total energy in the cylinder at the instant at which the next cycle begins.

The dynamics of change in the pressure, temperature, and Mach number during the cycle in the cylinder in the neighborhood of the valve is shown in Fig. 5a. The mass-mean values of the pressure and temperature over the period of the cycle in the low-pressure channel act as the normalization parameters. After the stage of admission of the gas, the diaphragm closes, with the result that the pressure and temperature decrease sharply and the gas in the neighborhood of the diaphragm becomes practically immobile. At the stage of release, it flows slowly. The diaphragm in the low-pressure channel closes at the instant of time at which the shock wave reflected from the piston surface arrives, which occurs with a sharp increase in the temperature and the pressure. The behavior of these gasdynamic quantities on the piston surface is shown in Fig. 5b. The quantities change stepwise when the shock wave arrives.

The distribution of the gasdynamic variables along the cylinder length and the dynamics of substantially nonstationary wave processes are shown in Fig. 6. The left edge of the curves in Fig. 6a corresponds to the parameters of the gas entering from the high-pressure channel. Under steady-state operating conditions, the gas is admitted at a resonance frequency. This is evidenced by the higher-than-average values of the pressure and by the nonmonotonic nature of temperature profile in the neighborhood of the valve. Figure 6b shows the state of the gasdynamic quantities by the moment at which the admission begins. Despite the fairly high average pressure and temperature in the cylinder, the values of these quantities in the neighborhood of the valve differ insignificantly from their values in the low-pressure channel.

Since the most important processes in the gas flowing through a wave expander proceed without a marked change in the entropy, it makes sense to compare the operating efficiency of the device studied starting from the operational characteristic of the reciprocating expander. If the operation of the reciprocating expander [2] is evaluated on the assumption that all the processes proceed isentropically (without loss), the change in the temperature is related to the change in the pressure by the formula

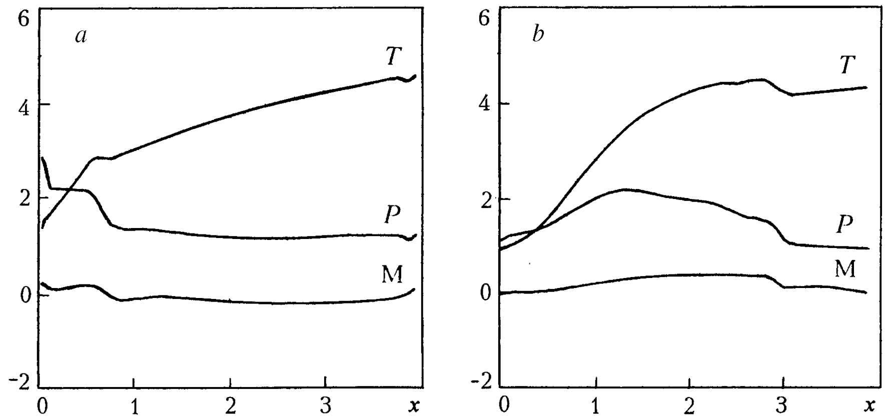


Fig. 6. Distribution of the pressure, temperature, and Mach number in the cylinder by the instant at which the admission (a) and release (b) begin.

$$\frac{T_{\text{in}}}{T_{\text{out}}} = \left(\frac{P_{\text{in}}}{P_{\text{out}}}\right)^{\frac{k-1}{k}}. \tag{5}$$

According to (5), if the ratio between the pressures in the high-pressure and low-pressure channels is equal to 4 and $k = 1.4$, the temperature ratio is 1.486. For the analogous evaluation of the operating efficiency of the wave expander in the form of the temperature ratio, it is necessary to calculate the mass-mean temperature at the exit from the wave expander. The results of the calculations carried out for the steady-state operating conditions show that the corresponding temperature ratio for the selected value of the pressure ratio in the high-pressure and low-pressure channels is equal to 1.333 for the wave expander. This allows the conclusion that the efficiency of the wave expander falls within the range of 0.7 to 0.8, i.e., it exceeds markedly the values characteristic of the above-mentioned nonequilibrium cryogenerators forming stationary or nonstationary wave structures and approaches the values characteristic of a reciprocating expander of the classical type.

NOTATION

t, time; τ, duration of the cycle; dt, differential with respect to time; H, total enthalpy; h, specific enthalpy; $E = H - P/\rho$, total energy; P, pressure; T, temperature; ρ, density; u, velocity; $U = [\rho; \rho u; \rho E]^{\mathrm{T}}$, vector of conservative variables; $F = [\rho(u - u_x); \rho(u - u_x)u; \rho(u - u_x)H + Pu_x]^{\mathrm{T}}$, vector of flows, written with allowance for the movement of the control surface with a velocity u_x; $u_{x\mathrm{p}}$, the velocity of movement of the piston; μ, molar mass of the gas; R, universal gas constant. Subscripts: in and out, values of the quantities at the entrance to and exit from the device; T, transposing; p, piston.

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